Dielectric function of a two-component plasma including collisions

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A multiple-moment approach to the dielectric function of a dense non-ideal plasma is treated beyond RPA including collisions in Born approximation. The results are compared with the perturbation expansion of the Kubo formula. Sum rules as well as Ward identities are considered. The relations to optical properties as well as to the dc electrical conductivity are pointed out.

I. INTRODUCTION

The dielectric function $\epsilon(\vec{k},\omega)$ is a physical quantity containing a lot of information about the plasma. In homogeneous, isotropic systems it is related to the electrical conductivity $\sigma(k,\omega)$ and the polarisation function $\Pi(k,\omega)$ according to

$$\epsilon(k,\omega) = 1 + \frac{i}{\epsilon_0 \omega}\,\sigma(k,\omega) = 1 - \frac{1}{\epsilon_0 k^2}\,\Pi(k,\omega). \tag{1}$$

A well established expression is the random phase approximation (RPA) valid for collisionless plasmas. The inclusion of collisions, however, is connected with difficulties. A perturbative treatment of the Kubo formula is not applicable near $\vec{k}=0$, $\omega=0$ because there is an essential singularity in zeroth order. Partial summations are sometimes in conflict with sum rules. Improvements of the RPA result are discussed in the static limit, where the local field corrections are treated in time-dependent mean-field theory [1]. Also approximations based on the sum rules for the lowest moments have been proposed [2]. However, an unambigous expression for $\epsilon(\vec{k},\omega)$ in the entire $\vec{k}\ \omega$ space cannot be given by these approaches.

A particular problem is the appropriate treatment of the long-wavelength limit $k \to 0$ at small frequencies where the dc conductivity should be obtained. In a previous paper [3] an approach has been given where this limiting case coincides with the Chapman-Enskog approach [4] to the dc conductivity. In particular, the polarization function was found as

$$\Pi(k,\omega) = i\,\frac{k^2}{\omega}\,\beta\,\Omega_0 \begin{vmatrix} 0 & M_{0n}(k,\omega) \\ M_{m0}(k,\omega) & M_{mn}(k,\omega) \end{vmatrix} \Big/ |M_{mn}(k,\omega)|. \tag{2}$$

The matrix elements M_{mn} are equilibrium correlation function which are explicitly given in the following section. They contain operators B_m and B_n which specify the nonequilibrium state.

For the evaluation of the dielectric function, we have to deal with two problems:
i) the choice of the operators B_n to describe the relevant fluctuations in the linear response regime,
ii) the evaluation of the equilibrium correlation functions.

The equilibrium correlation functions in a nonideal plasma can be evaluated using the method of thermodynamic Green functions. In lowest order of the perturbation theory to be considered here we have the Born approximation as described in [3]. Higher order terms can be taken into account in a systematic way, see [5].

With respect to the choice of the operators B_n, only the current density operator J has been considered in [3]. In the spirit of the Chapman-Enskog approach we will include here higher moments of the single-particle distribution function to study the convergency behavior. For the dc conductivity the answer is well known see [6]. Note that different approaches based on different sets of relevant observables B_n are formally equivalent as long as no approximations in evaluating the correlation functions are performed. However, within a finite order perturbation theory, the results for the conductivity are improved if the set of relevant observables is extended.

Results for the dielectric function within a four-moment approach are shown in Sec. II and compared with the results of a single-moment approach. Some exact relations are discussed in Sec. III. Of particular interest is the relation to the Kubo formula which may be treated in perturbation theory as discussed in Sec. IV.

II. FOUR-MOMENT APPROACH FOR A TWO-COMPONENT PLASMA

To evaluate the dielectric function we use the expression (2) for the polarisation function, where the matrix elements are given by

$$M_{0n}(k,\omega) = (J_k; B_n) \;, \qquad M_{m0}(k,\omega) = (B_m; \hat{J}_k) \;,$$
$$M_{mn}(k,\omega) = (B_m; [\dot{B}_n - i\omega B_n]) + \langle \dot{B}_m; [\dot{B}_n - i\omega B_n]\rangle_{\omega+i\eta} - \frac{\langle \dot{B}_m; J_k\rangle_{\omega+i\eta}}{\langle B_m; J_k\rangle_{\omega+i\eta}} \; \langle B_m; [\dot{B}_n - i\omega B_n]\rangle_{\omega+i\eta} \;. \tag{3}$$

The equilibrium correlation functions are defined as

$$(A;B) = (B^+;A^+) = \frac{1}{\beta}\int_0^\beta d\tau \; \mathrm{Tr}\left[A(-i\hbar\tau)B^+\rho_0\right] \;,$$
$$\langle A;B\rangle_z = \int_0^\infty dt \; \mathrm{e}^{izt} \, (A(t);B) \;, \tag{4}$$

with $A(t) = \exp(iHt/\hbar)\, A \, \exp(-iHt/\hbar)$ and $\dot{A} = \frac{i}{\hbar}[H,A]$. $\rho_0 = \exp(-\beta H + \beta\sum_c \mu_c N_c) \Big/ \mathrm{Tr}\exp(-\beta H + \beta \sum_c \mu_c N_c)$ is the equilibrium statistical operator.

We will consider a two-component plasma consisting of electrons ($c = e$) and ions ($c = i$). In particular, results are given below for a hydrogen plasma. With the single-particle operators

$$n^c_{p,k} = \left(n^c_{p,-k}\right)^+ = c^+_{p-k/2} \; c_{p+k/2} \tag{5}$$

the current density operator is given by

$$J_k = \frac{1}{\Omega_0} \sum_{c,p} \frac{e_c}{m_c} \; \hbar p_z \; n^c_{p,k} \;. \tag{6}$$

Furthermore we used the abbreviation $\hat{J}_k = \epsilon^{-1}(k,\omega)\; J_k$.

To select the relevant operators B_n, we restrict us to the ordinary kinetic approach. The inclusion of higher order correlations is also possible, see [5].

Within the kinetic approach, the nonequilibrium state of the plasma is described by the mean values of th single-particle operators (5) corresponding to an induced single-particle distribution function with wave number k. Instead of treating an infinit number of operators depending on the momentum p, we can restrict us to a finite number of moments of the distribution function. This procedure is familiar from the theory of the dc conductivity. Whereas in that case only moments with respect to p have to be selected, in the general case of arbitrary k to be considered here moments of p as well as $\vec{p}\cdot\vec{k}$ have to be taken into account.

In this paper we investigate how the lowest moment approach in Born approximation is modified if further moments are included. From the theory of dc conductivity we know that important modifications are obtained by including the energy current density in addition to the particle current density, i. e. if we include also $\vec{p}^2 p_z$. Then, the electrical cunductivity is not only described by the electron-ion interaction, but includes also the effects of electron-electron interaction which are not effective in the lowest moment approximation due to the conservation of total momentum.

The four-moment approach to be considered in this paper is given by the following moments of the electron ($c = e$) or ion ($c = i$) distribution function, respectively,

$$b^c_1(p) = \frac{\hbar}{\sqrt{2m_c kT}} p_z,$$
$$b^c_2(p) = \left(\frac{\hbar}{\sqrt{2m_c kT}}\right)^{3/2} (\vec{p})^2 p_z \;. \tag{7}$$

The evaluation of the corresponding correlation functions in Born approximation is given in the Appendix for the nondegenerate case. As a trivial result, in the lowest approximation with respect to the interaction the RPA result is recovered. In general the matrix elements are given in terms of integrals of expressions containing the Dawson integral.

To give an example, a hydrogen plasma is considered with parameter values $T = 98$ Ryd and $n_e = 8.9 \; a_B^{-3}$ which are found in the center of the sun [7]. The results are comparable to the results obtained in [3] for parameter values corresponding to laser produced high-density plasmas [8].

Results for the real and the imaginary part of the dielectric function in the two-moment approximation given by $b_1^c(p)$ are shown in figures 1 and 2, respectively. Besides the RPA dielectric function the one-moment calculation reported in [3] is shown as well. While the differences between the improved dielectric function and the RPA are small at high momenta ($k = 1\ a_B^{-1}$), significant changes occur at small momenta ($k = 0.1\ a_B^{-1}$). On the other hand, the one-moment approach is almost identical with the two-moment calculation. This is an indication that convergence is reached by augmenting the number of moments as is expected from earlier studies of the dc conductivity [6]. Note, that the static limit is given by the Debye law.

Results for the inverse dielectric function, which describes the response to the external potential, are shown in figures 3 ($k = 0.5\ a_B^{-1}$) and 4 ($k = 0.3\ a_B^{-1}$) and compared with the RPA inverse dielectric function. Major deviations occur only at frequencies close to the plasma frequency. For small momenta, the imaginary part of the dielectric function including collisions is considerably broader compared with the RPA one. While the imaginary part of the inverse dielectric function in the RPA approximation becomes delta-like in the long wavelength limit, a broadening of the plasmon peak appears, as can be seen from figure 4. Some properties of the dielectric function will be discussed in the following section.

III. EXACT RELATIONS FOR THE DIELECTRIC FUNCTION AND LIMITING CASES

Several exact properties of the dielectric function are known [9] such as sum rules

$$-\int_{-\infty}^{\infty} \frac{d\omega}{\pi}\, \omega \, \mathrm{Im}\, \epsilon^{-1}(k,\omega) = \omega_{pl}^2 \quad , \tag{8}$$

$$\int_{-\infty}^{\infty} \frac{d\omega}{\pi}\, \omega \, \mathrm{Im}\, \epsilon(k,\omega) = \omega_{pl}^2 \quad , \tag{9}$$

$$\lim_{k\to 0} \int_{-\infty}^{\infty} \frac{d\omega}{\pi}\, \frac{1}{\omega} \, \mathrm{Im}\, \epsilon^{-1}(k,\omega) = -1 \quad , \tag{10}$$

the long-wavelength limit

$$\lim_{k\to 0} \mathrm{Re}\, \epsilon(k,0) = 1 + V(k)\, n^2\, K \quad . \tag{11}$$

Here $\omega_{pl}^2 = \sum_{c=e,i} \left(e^2\, n_c\right) / \left(\epsilon_0\, m_c\right)$ denotes the plasma frequency and K the isothermal compressibility. Further extensions for a two-component system can be found in [10]. This is a special relation resulting from the relation between the dynamical structure factor

$$S(k,\omega) = \frac{1}{2\,\pi} \int_{-\infty}^{\infty} dt\, < \rho_k^+(t)\, \rho_k > \, e^{i\omega t} \tag{12}$$

and the dielectric function which can be established via the fluctuation-dissipation-theorem

$$S(k,\omega) = -\frac{1}{\pi}\, \frac{1}{e^{\beta\,\omega} - 1}\, \mathrm{Im}\, \epsilon^{-1}(k,\omega^-) \quad . \tag{13}$$

Furthermore, the Kramers-Kronig relation holds which connects the real and the imaginary part of the dielectric function:

$$\mathrm{Re}\epsilon(k,\omega) = 1 \,+\, P \int \frac{d\omega'}{\pi}\, \frac{\mathrm{Im}\, \epsilon(\vec{k},\omega')}{\omega - \omega'} \ . \tag{14}$$

Here, P denotes the Cauchy principal value integration. The inverse dielectric function obeys a corresponding relation. Combining the Kramers-Kronig relation with the sum rules results in rigorous statements about the asymptotic behaviour at high frequencies:

$$\lim_{\omega\to\infty} \mathrm{Re}\epsilon(k,\omega) = 1 - \frac{\omega_{pl}^2}{\omega^2} + O(\frac{1}{\omega^4}) \quad . \tag{15}$$

We test the two-moment approach by checking the sum rules as well as the asymptotic behaviour. It is found that the sum rules are fulfilled within the numerical accuracy ($\approx 0.1\%$). The Kramers-Kronig relation holds as well.

Having in mind relation (1), the dc conductivity can be obtained considering the limitng case $k \to 0$. A comparison with other results for the conductivity can be made by parameterising the conductivity via

$$\sigma = \sigma(0,0) = s \frac{(k_B\,T)^{3/2}\,(4\pi\epsilon_0)^2}{e^2\,m_{ei}^{1/2}}\,\frac{1}{\Phi} \quad , \tag{16}$$

where Φ denotes the Coulomb-logarithm and m_{ei} the reduced mass of the electron.

As shown in figure 5, there is no shift of the maximum of the inverse dielectric function, while the plasmon peak is broadened. Moreover, the long wavelength limit can be described by a Drude-like formula, implying that the width of the plasmon peak is given by the dc conductivity. The form of the plasmon peak can be compared with computer simulation studies. In contrast to RPA calculations, width as well as height of the plasmon peak in our calculation are of comparable size as computer simulations [12].

IV. COMPARISON WITH THE KUBO FORMULA

Depending on the selected set of relevant operators $\{B_n\}$, different expressions for the dielectric function can be derived within linear response theory. A often used expression is the Kubo formula [13] as given by

$$\Pi(k,\omega) = -\,\frac{ik^2\beta\Omega_0}{\omega}\,\langle J_k;\hat{J}_k\rangle_{\omega+i\eta}\ . \tag{17}$$

As shown in [3], this result follows as a special case within the generalized linear response theory. As also shown there, the different expressions identical in the limit $\eta \to 0$ if no further approximations are performed.

The advantage of linear response theory is that the evaluation of the dielectric function is related to the evaluation of equilibrium correlation functions. In dense, strongly coupled systems, these correlation functions can be calculated with computer simulations. Another possibility is to use peturbation theory which is most effectively formulated with the concept of thermodynamic Green functions [14].

In zeroth order with respect to the interaction, from (17) immediately the RPA result is obtained, in coincidence with all other approaches including J within the set of relevant operators. The first order expansion with respect to the screened interaction reads

$$\begin{aligned}
\Pi(k,\omega_\lambda) &= \sum_p \left(f_p + f_p' n_{\rm ion}\sum_q V_q^2 \frac{1}{E_p - E_{p-q}}\right)\left(\frac{1}{E_p-\omega_\lambda-E_{p-k}} + \frac{1}{E_p+\omega_\lambda-E_{p+k}}\right)\\
&+ n_{\rm ion}\sum_{pq} V_q^2 f_p \frac{kq}{m}\frac{1}{E_p-E_{p-q}}\frac{1}{E_p-\omega_\lambda-E_{p-k}}\frac{1}{E_p-\omega_\lambda-E_{p-k-q}}\left(\frac{1}{E_p-E_{p-q}}+\frac{1}{E_p-\omega_\lambda-E_{p-k}}\right)\\
&+ (\omega,k \leftrightarrow -\omega,-k).
\end{aligned} \tag{18}$$

For the sake of simplicity, we have taken the adiabatic limit where $m_i/m_e \to \infty$ (Lorentz plasma), In particular we find for $k \to 0$

$${\rm Im}\Pi(k,\omega) = n\sum_{pq} V_q^2\left(\frac{kq}{m}\right)^2 \pi\delta(E_p-\omega-E_{p-q}){\rm e}^{-\beta(E_p-\mu)}\frac{1-{\rm e}^{\beta\omega}}{\omega^4}. \tag{19}$$

what gives the frequency-dependent conductivity.

However, this perturbation expansion does not converge at $\omega \to 0$, and partial summations have to be performed. For instance, a simple approximation for the polarization function including interactions with further particles would be a polarization function given by the product of two full propagators. This way, the polarization function contains shifts and damping of the single-particle states due to the interaction with the medium. However, this approximation does not fulfill rigorous relations such as sum rules, since important corrections to the RPA of the same order in the density as the considered ones are missing, e.g. vertex corrections. These corrections are linked to the self-energy by Ward identities [15]. As a consequence, the vertex has to be improved in accordance with the self-energy. Following Baym and Kadanoff [16], a consistent vertex can be constructed to a given self-energy. However, the solution of the vertex equation cannot be given in a simple algebraic form, and usually some approximations are performed, see [17].

V. CONCLUSIONS

An approach to the dielectric function has been investigated which includes the effects of collissions and can be used in the entire k, ω space. Within a four-moment approach to a two-component plasma, the Born approximation has been evaluated, and important rigorous results for the dielectric function are checked. Compared with the ordinary Kubo formula, the approach given here seems to be more appropriate for perturbation expansions.

In particular, comparing with a one-moment approach, the convergency behavior of this method was inspected. As well known from the theory of dc conductivity, convergence is expected if higher moments are included. In a more general approch, also two-particle correlations can be included into the set of relevant operators.

Within a quantum statistical approach, the Born approximation can be improved by systematic treatment of Green functions. This concerns, e.g., the inclusion of strong collision by treating T-matrices, degeneracy effects, and the treatment of the dynamic screening of the interaction. Here, the comparison with computer simulations is also an interesting perspective. Work in this direction is in progress.

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APPENDIX: EVALUATION OF THE MATRIX ELEMENTS OF Π

We start from the general expression (1), (2) for the dielectric function with ($n, m =$1...4, $\quad c, d = e, i$)

$$M^d_{0m}(q,\omega) = \frac{1}{\Omega_0} \sum_{c,p,k} \frac{e_c}{m_c} \frac{q}{\omega} \hbar p_z b^d_m(k) (n^d_{k,-q}; n^c_{p,q}) \ , \tag{20}$$

$$M^c_{n0}(q,\omega) = \frac{1}{\Omega_0} \sum_{d,p,k} iq \frac{e_d}{m_d} \hbar k_z [b^c_n(p)]^* (n^d_{k,-q}; n^c_{p,q}) \ , \tag{21}$$

$$M^{cd}_{nm}(q,\omega) = \frac{1}{\Omega_0} \sum_{p,k} [b^c_n(p)]^* b^d_m(k) ([\dot{n}^d_{k,-q} - i\omega n^d_{k,-q}]; n^c_{p,q}) + A \tag{22}$$

or, after some rearrangements,

$$\epsilon(q,\omega) = 1 - \frac{\beta n e^2}{\epsilon_0 q \omega} \begin{vmatrix} 0 & \tilde{M}^d_{0m}(q,\omega) \\ \tilde{M}^c_{n0}(q,\omega) & \tilde{M}^{cd}_{nm}(q,\omega) \end{vmatrix} / |\tilde{M}^{cd}_{nm}(q,\omega)| \tag{23}$$

with

$$\tilde{M}^d_{0m}(q,\omega) = \frac{z_d}{ne} M^d_{0m}(q,\omega) = \frac{z_d}{ne} \frac{1}{\Omega_0} \sum_{c,p,k} \frac{e_c}{m_c} \frac{q}{\omega} \hbar p_z b^d_m(k) (n^d_{k,-q}; n^c_{p,q}) \ , \tag{24}$$

$$\tilde{M}^c_{n0}(q,\omega) = -i \frac{1}{\omega} \frac{z_c}{ne} M^c_{n0}(q,\omega) = -i \frac{1}{\omega} \frac{z_c}{ne} \frac{1}{\Omega_0} \sum_{d,p,k} iq \frac{e_d}{m_d} \hbar k_z [b^c_n(p)]^* (n^d_{k,-q}; n^c_{p,q}) \ , \tag{25}$$

$$\begin{aligned} \tilde{M}^{cd}_{nm}(q,\omega) &= -i \frac{\sqrt{m_c m_d}}{2kTnq} M^{cd}_{nm}(q,\omega) = -i \frac{\sqrt{m_c m_d}}{2kTnq} \frac{1}{\Omega_0} \sum_{p,k} [b^c_n(p)]^* b^d_m(k) \left\{ ([\dot{n}^d_{k,-q} - i\omega n^d_{k,-q}]; n^c_{p,q}) \right. \\ &\left. + \langle \dot{n}^d_{k,-q}; [\dot{n}^c_{p,q} - i\omega n^c_{p,q}] \rangle_{\omega+i\eta} - \frac{\langle \dot{n}^d_{k,-q}; J_k \rangle_{\omega+i\eta}}{\langle n^d_{k,-q}; J_k \rangle_{\omega+i\eta}} \langle n^d_{k,-q}; [\dot{n}^c_{p,q} - i\omega n^c_{p,q}] \rangle_{\omega+i\eta} \right\} \ , \end{aligned} \tag{26}$$

and

$$z_c = \frac{\omega}{q}\sqrt{\frac{m_c}{2kT}} \ . \tag{27}$$

We specify to a four-moment approach (7) where $B_1 = b_1^e(p),\ B_2 = b_2^e(p),\ B_3 = b_1^i(p),\ B_4 = b_1^i(p)$. Introducing the Dawson integral

$$D(z) = \lim_{\delta\to+0} \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} dx \mathrm{e}^{-x^2} \frac{1}{x-z-i\delta} \tag{28}$$

and using the abbreviations

$$r_1^c = \frac{1}{2}\ \frac{1}{1+z_c D(z_c)} \ , \tag{29}$$

$$r_2^c = \frac{5}{4}\ \frac{1}{0.5+(1+z_c^2)[1+z_c D(z_c)]} \ , \tag{30}$$

we have for $n_e = n_i = n$, $e_e = -e_i = e$

$$\tilde{M}_{01}^e = \frac{1}{2}, \qquad \tilde{M}_{02}^e = \frac{5}{4}, \qquad \tilde{M}_{03}^i = -\frac{1}{2}, \qquad \tilde{M}_{04}^i = -\frac{5}{4}, \tag{31}$$

and

$$\tilde{M}_{10}^e = \frac{1}{2}, \qquad \tilde{M}_{20}^e = \frac{5}{4}, \qquad \tilde{M}_{30}^i = -\frac{1}{2}, \qquad \tilde{M}_{40}^i = -\frac{5}{4}, \tag{32}$$

We decompose

$$\tilde{M}_{nm}^{cd}(q,\omega) = a_{nm} + b_{nm} + c_{nm} \tag{33}$$

and find in zeroth order with respect to the interaction

$$a_{11} = \frac{1}{2} r_1^e \frac{q}{\omega}, \qquad a_{12} = \frac{5}{4} r_1^e \frac{q}{\omega}, \qquad a_{21} = \frac{5}{4} r_2^e \frac{q}{\omega}, \qquad a_{22} = \frac{35}{8} r_2^e \frac{q}{\omega}, \tag{34}$$

and

$$a_{33} = \frac{1}{2} r_1^i \frac{q}{\omega}, \qquad a_{34} = \frac{5}{4} r_1^i \frac{q}{\omega}, \qquad a_{43} = \frac{5}{4} r_2^i \frac{q}{\omega}, \qquad a_{44} = \frac{35}{8} r_2^i \frac{q}{\omega}. \tag{35}$$

The b_{nm} contain the electron-ion interaction in first Born approximation and the c_{nm} the electron-electron or ion-ion interaction, respectively. We use a screened interaction with the Debye screening factor $\exp(-\kappa r)$, $\kappa^2 = \sum_c n_c e_c^2/(\epsilon_0 kT)$.

Terms due to electron-ion interaction are with $M = m_e + m_i$

$$b_{ij} = -\frac{i}{8(2\pi)^{3/2}} \frac{1}{q} n \frac{e^4}{\epsilon_0^2} \left(\frac{1}{kT}\right)^{5/2} \left(\frac{m_e m_i}{M}\right)^{1/2} g_{ij} \ . \tag{36}$$

With

$$z_{ei} = \frac{\omega}{q}\sqrt{\frac{M}{2kT}} \tag{37}$$

and

$$\lambda^{ei} = 1 + \frac{\hbar^2\kappa^2 M}{4 m_e m_i kT}\,\frac{1}{p^2} \ , \tag{38}$$

$$\Lambda_1 = [\ln\left(\frac{\lambda^{ei}-1}{\lambda^{ei}+1}\right) + \frac{2}{\lambda^{ei}+1}] \ , \tag{39}$$

$$\Lambda_2 = [\lambda^{ei} \ln \left(\frac{\lambda^{ei} - 1}{\lambda^{ei} + 1} \right) + 2], \qquad \Lambda_3 = \frac{2}{(\lambda^{ei})^2 - 1} \ , \tag{40}$$

$$R_n^c = \frac{M}{\sqrt{m_e m_i}} r_n^c \ , \tag{41}$$

$$D_e = D(z_{ei} - \sqrt{\frac{m_i}{m_e}} pc), \qquad D_i = D(z_{ei} - \sqrt{\frac{m_e}{m_i}} pc) \ , \tag{42}$$

we find

$$g_{11} = \int_0^\infty dp \mathrm{e}^{-p^2} \, \Lambda_1 \left\{ \frac{2}{3} p - R_1^e \int_{-1}^1 dc \, c(2D_e + D_i) \right\} \ , \tag{43}$$

$$g_{13} = \int_0^\infty dp \mathrm{e}^{-p^2} \, \Lambda_1 \left\{ -\frac{2}{3} p + R_1^e \int_{-1}^1 dc \, c D_e \right\} \ , \tag{44}$$

$$\begin{aligned} g_{12} = \int_0^\infty dp \mathrm{e}^{-p^2} \ & \Big\{ \Lambda_1 (\frac{5}{3} \frac{m_e}{M} p + \frac{2}{3} \frac{m_i}{M} p^3 + R_1^e 2 \frac{\sqrt{m_e m_i}}{M} p) + \\ & + \int_{-1}^1 dc R_1^e (D_e \left[\Lambda_1 c(-\frac{5}{2} - \frac{m_e}{M} - \frac{m_i}{M} p^2 - 3 \frac{m_e}{M} (z_{ei} - \sqrt{\frac{m_i}{m_e}} pc)^2) + \Lambda_2 2p \frac{\sqrt{m_e m_i}}{M} (1 - 3c^2)(z_{ei} - \sqrt{\frac{m_i}{m_e}} pc) \right] \\ & + \ D_i \Lambda_1 c(-\frac{5}{2})) \Big\} \ , \end{aligned} \tag{45}$$

$$\begin{aligned} g_{14} = \int_0^\infty dp \mathrm{e}^{-p^2} \ & \Big\{ \Lambda_1 (-\frac{5}{3} \frac{m_i}{M} p - \frac{2}{3} \frac{m_e}{M} p^3 - R_1^e 2 \frac{\sqrt{m_e m_i} m_i}{M m_e} p) + \\ & + \int_{-1}^1 dc R_1^e (D_e \left[\Lambda_1 c(\frac{m_i}{M} + \frac{m_e}{M} p^2 + 3 \frac{m_i}{M} (z_{ei} - \sqrt{\frac{m_i}{m_e}} pc)^2) + \Lambda_2 2p \frac{\sqrt{m_e m_i}}{M} (1 - 3c^2)(z_{ei} - \sqrt{\frac{m_i}{m_e}} pc) \right]) \Big\} \ , \end{aligned} \tag{46}$$

$$\begin{aligned} g_{21} = \int_0^\infty dp \mathrm{e}^{-p^2} \ & \Big\{ \Lambda_1 p (\frac{5}{3} \frac{m_e}{M} + \frac{2}{3} \frac{m_i}{M} p^2 + R_2^e (\frac{4}{3} \frac{\sqrt{m_e m_i}}{M} + 2 \frac{m_e \sqrt{m_e m_i}}{m_i M})) + \int_{-1}^1 dc R_2^e (D_e \\ & \times \left[\Lambda_1 c(-2 \frac{m_e}{M} - 2 \frac{m_i}{M} p^2 - 2c \frac{\sqrt{m_e m_i}}{M} p(z_{ei} - \sqrt{\frac{m_i}{m_e}} pc) - 4 \frac{m_e}{M} (z_{ei} - \sqrt{\frac{m_i}{m_e}} pc)^2) + \Lambda_2 2p \frac{\sqrt{m_e m_i}}{M} (1 - 3c^2)(z_{ei} - \sqrt{\frac{m_i}{m_e}} pc) \right] \\ & + \ D_i \left[\Lambda_1 c(-\frac{m_e}{M} - \frac{m_i}{M} p^2 - 3 \frac{m_e}{M} (z_{ei} - \sqrt{\frac{m_e}{m_i}} pc)^2) - \Lambda_2 2p \frac{\sqrt{m_e m_i}}{M} (1 - 3c^2)(z_{ei} - \sqrt{\frac{m_e}{m_i}} pc) \right]) \Big\} \ , \end{aligned} \tag{47}$$

$$\begin{aligned} g_{23} = \int_0^\infty dp \mathrm{e}^{-p^2} \ & \Big\{ \Lambda_1 p \left(-\frac{5}{3} \frac{m_e}{M} - \frac{2}{3} \frac{m_i}{M} p^2 + R_2^e \frac{2}{3} \frac{\sqrt{m_e m_i}}{M} \right) + \\ & + \int_{-1}^1 dc R_2^e D_e \Lambda_1 c \left[\frac{m_e}{M} + \frac{m_i}{M} p^2 + 2c \frac{\sqrt{m_e m_i}}{M} p(z_{ei} - \sqrt{\frac{m_i}{m_e}} pc) + \frac{m_e}{M} (z_{ei} - \sqrt{\frac{m_i}{m_e}} pc)^2 \right] \Big\} \ , \end{aligned} \tag{48}$$

$$\begin{aligned} g_{22} = \int_0^\infty dp \mathrm{e}^{-p^2} \ & \Big\{ \Lambda_1 (\frac{47}{6} \frac{m_e^2}{M^2} p + \frac{10}{3} \frac{m_e m_i}{M^2} p^3 + \frac{2}{3} \frac{m_i^2}{M^2} p^5 \\ & + R_2^e \frac{\sqrt{m_e m_i}}{M} (-\frac{10}{3} \frac{m_e}{M} p + 7 \frac{m_e}{m_i} p + 7p + \frac{11}{3} \frac{m_e}{M} p) + 2 \frac{m_e}{M} z_{ei}^2 p + \frac{2}{15} \frac{m_i}{M} p^3) \\ & + \Lambda_2 (\frac{8}{15} \frac{m_e^2}{M^2} p + \frac{40}{15} \frac{m_e m_i}{M^2} p^3 + R_2^e \frac{\sqrt{m_e m_i}}{M} (-\frac{16}{15} \frac{m_e}{M} p + \frac{16}{15} \frac{m_i}{M} p^3)) + \Lambda_3 (-\frac{4}{15} \frac{m_e^2}{M^2} p + R_2^e \frac{\sqrt{m_e m_i}}{M} \frac{8}{15} \frac{m_e}{M} p)) \end{aligned}$$

$$
\begin{aligned}
&+\int_{-1}^{1} dcR_2^e(D_e\Big[-\Lambda_1 c(\frac{7}{2}\frac{m_i}{M}p^2+\frac{m_i^2}{M^2}p^4+2c\frac{m_i}{M}\frac{\sqrt{m_em_i}}{M}p^3(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp)+\frac{7}{2}\frac{m_e}{M}(1+3(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp)^2)\\
&+2\frac{m_em_i}{M^2}p^2(1+2(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp)^2)+2c\frac{m_e}{M}\frac{\sqrt{m_em_i}}{M}p(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp)[1+3(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp)^2]\\
&+\frac{m_e^2}{M^2}[2+4(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp)^2+3(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp)^4])+\Lambda_2\frac{\sqrt{m_em_i}}{M}p\\
&\times(-6c(1-c^2)p\frac{\sqrt{m_em_i}}{M}-2(1-c^2)(1-3c^2)\frac{m_e}{M}(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp)+(1-3c^2)(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp)[7+2\frac{m_i}{M}p^2\\
&+4cp\frac{\sqrt{m_em_i}}{M}(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp)+2\frac{m_e}{M}(1+(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp)^2)])\\
&+2\Lambda_3\frac{\sqrt{m_em_i}}{M}\frac{m_e}{M}pc^2(1-c^2)(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp)\Big]\\
&+\ D_i\Big[\Lambda_1 c(-\frac{7}{2}\frac{m_e}{M}-\frac{7}{2}\frac{m_i}{M}p^2-\frac{21}{2}\frac{m_e}{M}(z_{ei}-\sqrt{\frac{m_e}{m_i}}pc)^2)-\Lambda_2 7p\frac{\sqrt{m_em_i}}{M}(1-3c^2)(z_{ei}-\sqrt{\frac{m_e}{m_i}}pc)\Big])\Big\}\ ,
\end{aligned}
\tag{49}
$$

$$
\begin{aligned}
g_{24}=&\int_0^\infty dp e^{-p^2}\Big\{\Lambda_1 p(-\frac{47}{6}\frac{m_em_i}{M^2}-\frac{5}{3}\frac{m_e^2}{M^2}p^2-\frac{5}{3}\frac{m_i^2}{M^2}p^2-\frac{2}{3}\frac{m_em_i}{M^2}p^4\\
&+R_2^e\frac{\sqrt{m_em_i}}{M}(-\frac{1}{3}\frac{m_i}{M}-2\frac{m_i}{M}z_{ei}^2+\frac{2}{3}\frac{m_e}{M}p^2-\frac{4}{5}\frac{m_i^2}{m_eM}p^2)\\
&+\Lambda_2 p(-\frac{8}{15}\frac{m_em_i}{M^2}+\frac{40}{15}\frac{m_em_i}{M^2}p^2+\frac{16}{15}\frac{\sqrt{m_em_i}}{M}\frac{m_i}{M}R_2^e(1+p^2))+\Lambda_3 p(\frac{4}{15}\frac{m_em_i}{M^2}-R_2^e\frac{\sqrt{m_em_i}}{M}\frac{8}{15}\frac{m_i}{M})\\
&+\int_{-1}^{1} dcR_2^e(D_e\Big[\Lambda_1 c\frac{\sqrt{m_em_i}}{M}(\frac{\sqrt{m_em_i}}{M}p^4+2c\frac{m_e}{M}p^3(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp)+\sqrt{\frac{m_e}{m_i}}\frac{m_e}{M}p^2(1+(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp)^2)\\
&+\sqrt{\frac{m_i}{m_e}}\frac{m_i}{M}p^2(1+3(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp)^2)+2c\frac{m_i}{M}p(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp)[1+3(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp)^2]\\
&+\frac{\sqrt{m_em_i}}{M}[2+4(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp)^2+3(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp)^4])\\
&+\Lambda_2\frac{\sqrt{m_em_i}}{M}[6c(-1+c^2)\frac{\sqrt{m_em_i}}{M}p^2+2(1-c^2)(1-3c^2)\frac{m_i}{M}p(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp)\\
&+(1-3c^2)p(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp)(2\frac{m_i}{M}p^2+4cp\frac{\sqrt{m_em_i}}{M}(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp)+2\frac{m_e}{M}(1+(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp)^2))]\\
&-\ \Lambda_3\frac{\sqrt{m_em_i}}{M}\frac{m_i}{M}2c^2(1-c^2)p(z_{ei}-\sqrt{\frac{m_i}{m_e}}cp)\Big])\Big\}\ .
\end{aligned}
\tag{50}
$$

The remaining expressions ($i = 3, 4$) follow as

$$
\begin{aligned}
&g_{31}=[g_{13},(e\leftrightarrow i)], \quad g_{32}=[g_{14},(e\leftrightarrow i)], \quad g_{33}=[g_{11},(e\leftrightarrow i)], \quad g_{34}=[g_{12},(e\leftrightarrow i)],\\
&g_{41}=[g_{23},(e\leftrightarrow i)], \quad g_{42}=[g_{24},(e\leftrightarrow i)], \quad g_{43}=[g_{21},(e\leftrightarrow i)], \quad g_{44}=[g_{22},(e\leftrightarrow i)].
\end{aligned}
\tag{51}
$$

For the collisions between identical species (e, i) we have

$$
c_{ij}^c=-\frac{i}{8(2\pi)^{3/2}}\frac{1}{q}n\frac{e^4}{\epsilon_0^2}\left(\frac{1}{kT}\right)^{5/2}\left(\frac{m_c}{2}\right)^{1/2}h_{ij}
\tag{52}
$$

and

$$
\lambda^c=1+\frac{\hbar^2\kappa^2}{2m_ckT}\frac{1}{p^2}
\tag{53}
$$

so that the contributions of electron-electron collisions ($i, j = 1, 2$) follow as

$$
h_{11}=0\ ,
\tag{54}
$$

$$h_{12} = 4r_1^e \int_0^\infty dp \int_{-1}^1 dc \mathrm{e}^{-p^2}\, p(1-3c^2)[\lambda^e \ln\left(\frac{\lambda^e-1}{\lambda^e+1}\right)+2](\sqrt{2}z_e-cp)D(\sqrt{2}z_e-cp)\ , \tag{55}$$

$$h_{21} = 4r_2^e \int_0^\infty dp \int_{-1}^1 dc \mathrm{e}^{-p^2}\, p(1-3c^2)[\lambda^e \ln\left(\frac{\lambda^e-1}{\lambda^e+1}\right)+2](\sqrt{2}z_e-cp)D(\sqrt{2}z_e-cp)\ , \tag{56}$$

$$\begin{aligned} h_{22} = &\int_0^\infty dp \mathrm{e}^{-p^2}\, [\lambda^e \ln\left(\frac{\lambda^e-1}{\lambda^e+1}\right)+2]\left\{\frac{4}{3}p^3+\frac{16}{15}r_2^e p^3+2r_2^e\int_{-1}^1 dc\, p\right. \\ &\times \left.[(1-3c^2)(\sqrt{2}z_e-cp)(p^2-p^2c^2+8+2z_e^2)+3pc(c^2-1)]D(\sqrt{2}z_e-cp)\right\}\ . \end{aligned} \tag{57}$$

The expressions for ion-ion collisions $(i,j=3,4)$ follow as

$$h_{33}=[h_{11},(e\leftrightarrow i)],\qquad h_{34}=[h_{12},(e\leftrightarrow i)],\qquad h_{43}=[h_{21},(e\leftrightarrow i)],\qquad h_{44}=[h_{22},(e\leftrightarrow i)], \tag{58}$$

i.e. replacing the index e in $c_{ij}^e,\ \lambda^e,\ z_e$ by the index i.

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Figure Captions:

Fig. 1:
Real and imaginary part of the dielectric function as a function of the frequency at fixed wavenumber $k = 1\ a_B^{-1}$. The two-moment approach is compared with the one-moment approach and the RPA.

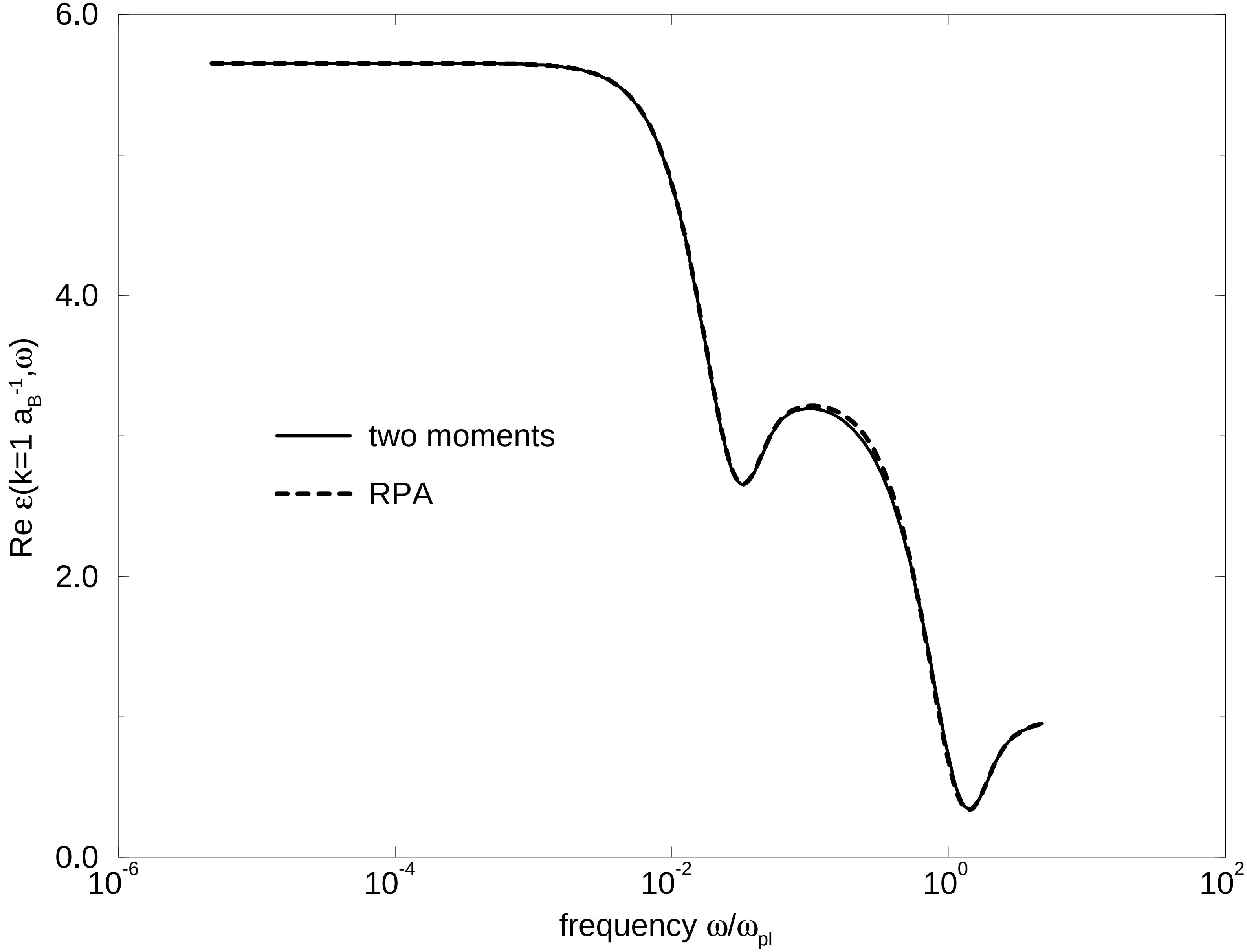
Fig. 2:
The same as Fig. 1 for wavenumber $k = 0.1\ a_B^{-1}$.

Fig. 3:
Imaginary part of the inverse dielectric function as a function of the frequency at fixed wavenumber $k = 0.5\ a_B^{-1}$. The two-moment approach is compared with the RPA.

Fig. 4:
The same as Fig. 3 for wavenumber $k = 0.3\ a_B^{-1}$.

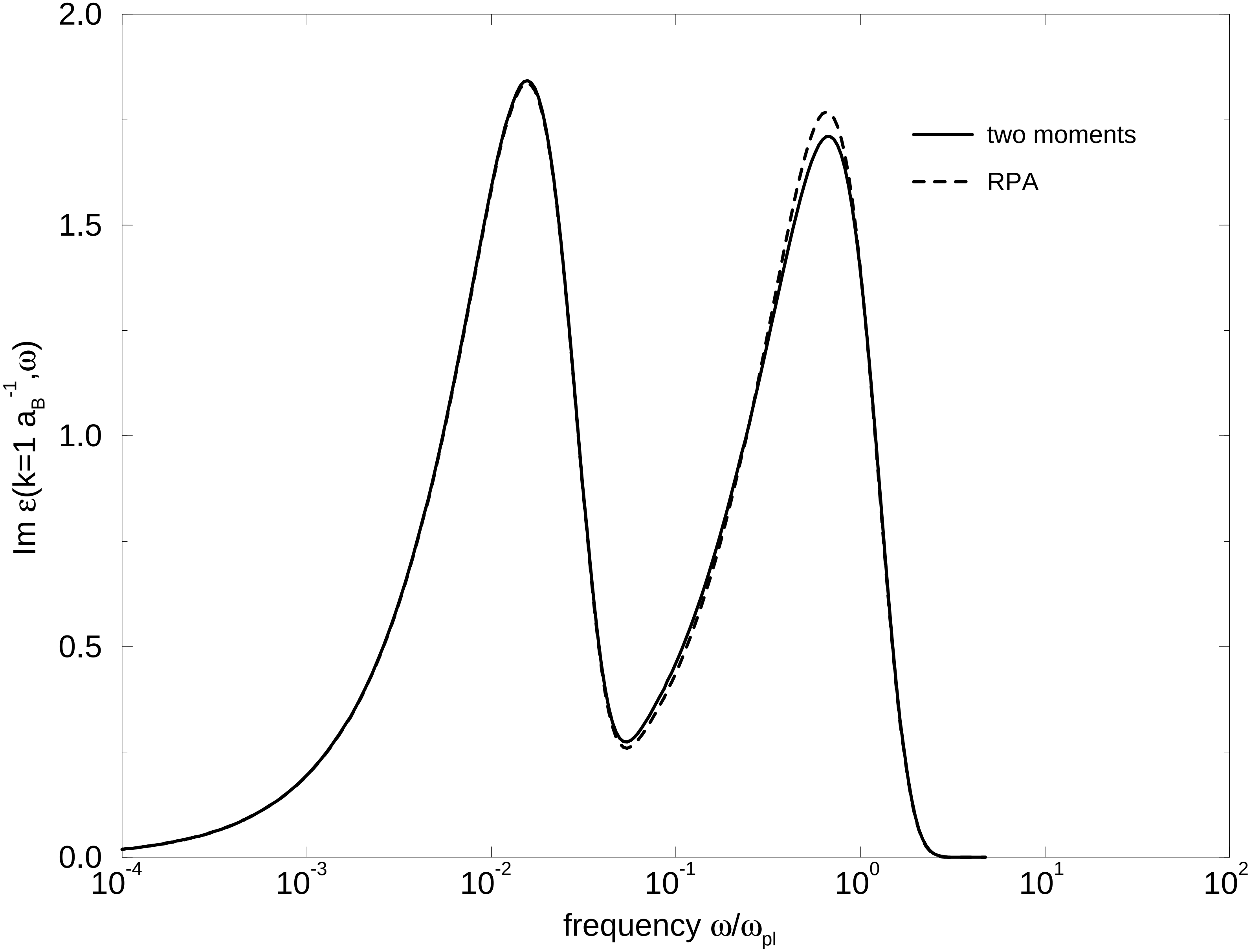
Fig. 5:
Imaginary part of the inverse dielectric function as a function of the frequency at different wavenumbers.

Fig. 1a



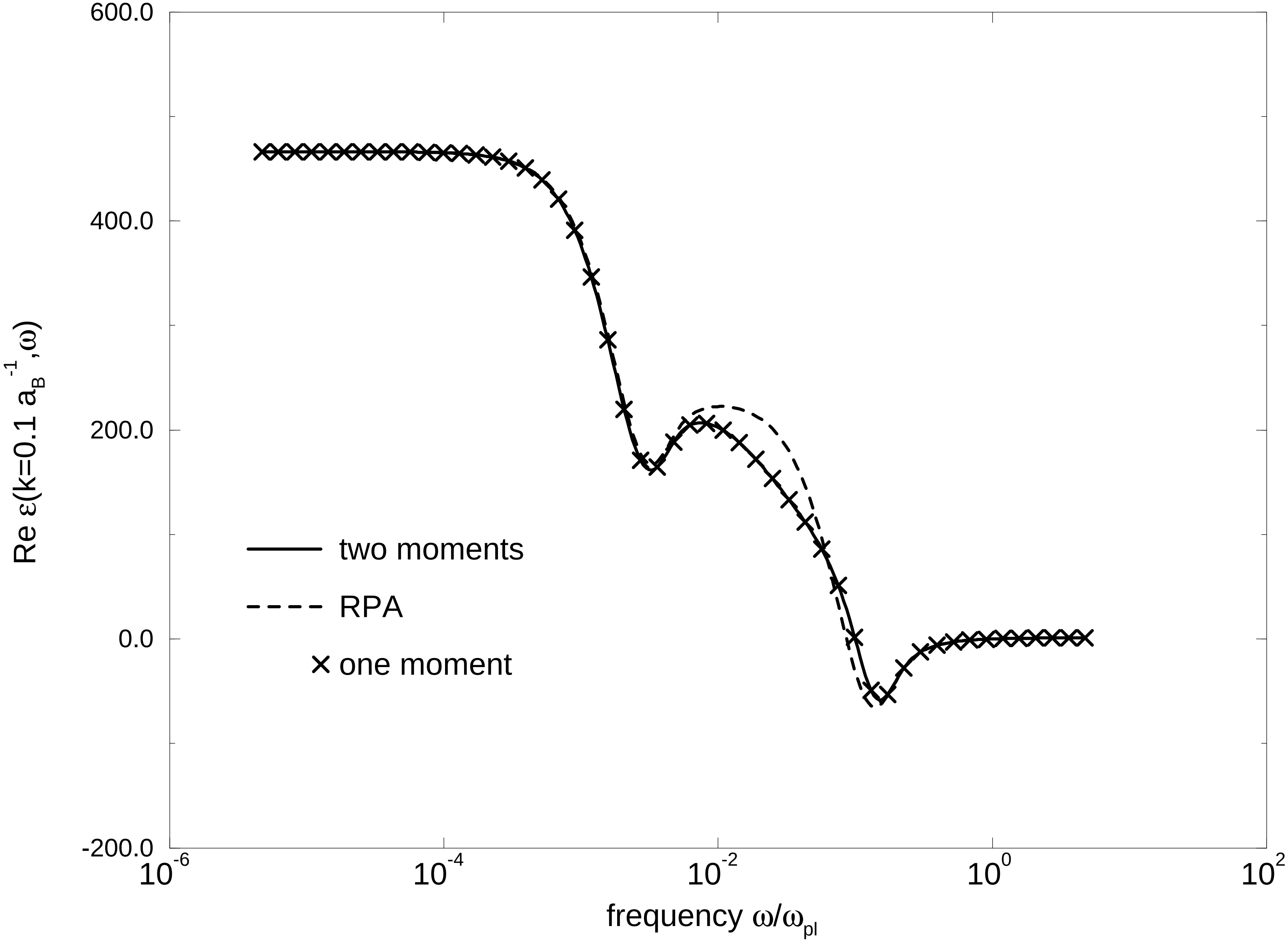
6.0
4.0
2.0
0.0
Re ε(k=1 a_B^-1,ω)
two moments
RPA
10^-6
10^-4
10^-2
10^0
10^2
frequency ω/ω_pl

Fig. 1b



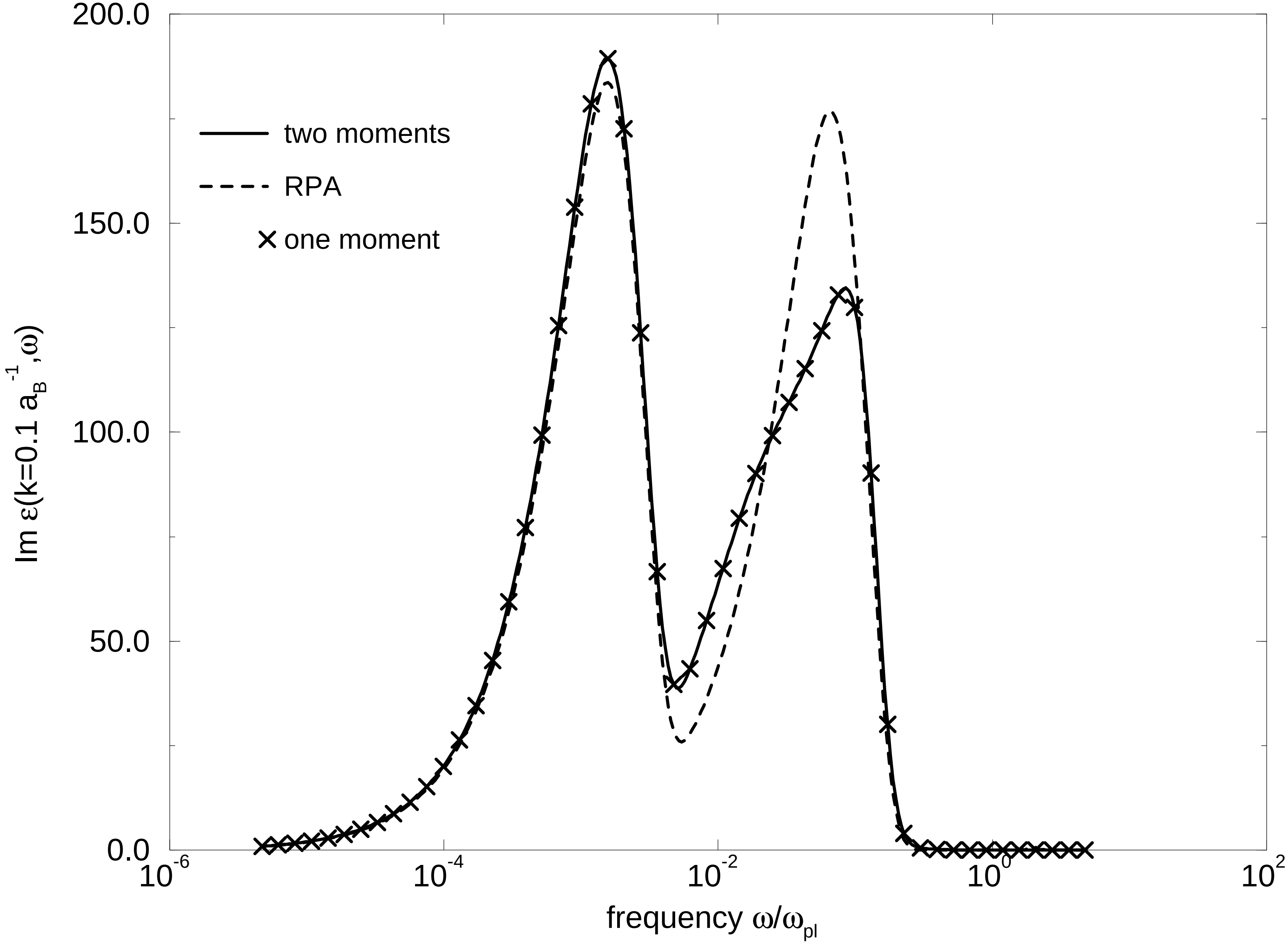
2.0
1.5
1.0
0.5
0.0
Im ε(k=1 a_B^-1,ω)
10^-4
10^-3
10^-2
10^-1
10^0
10^1
10^2
frequency ω/ω_pl
two moments
RPA

Fig. 2a



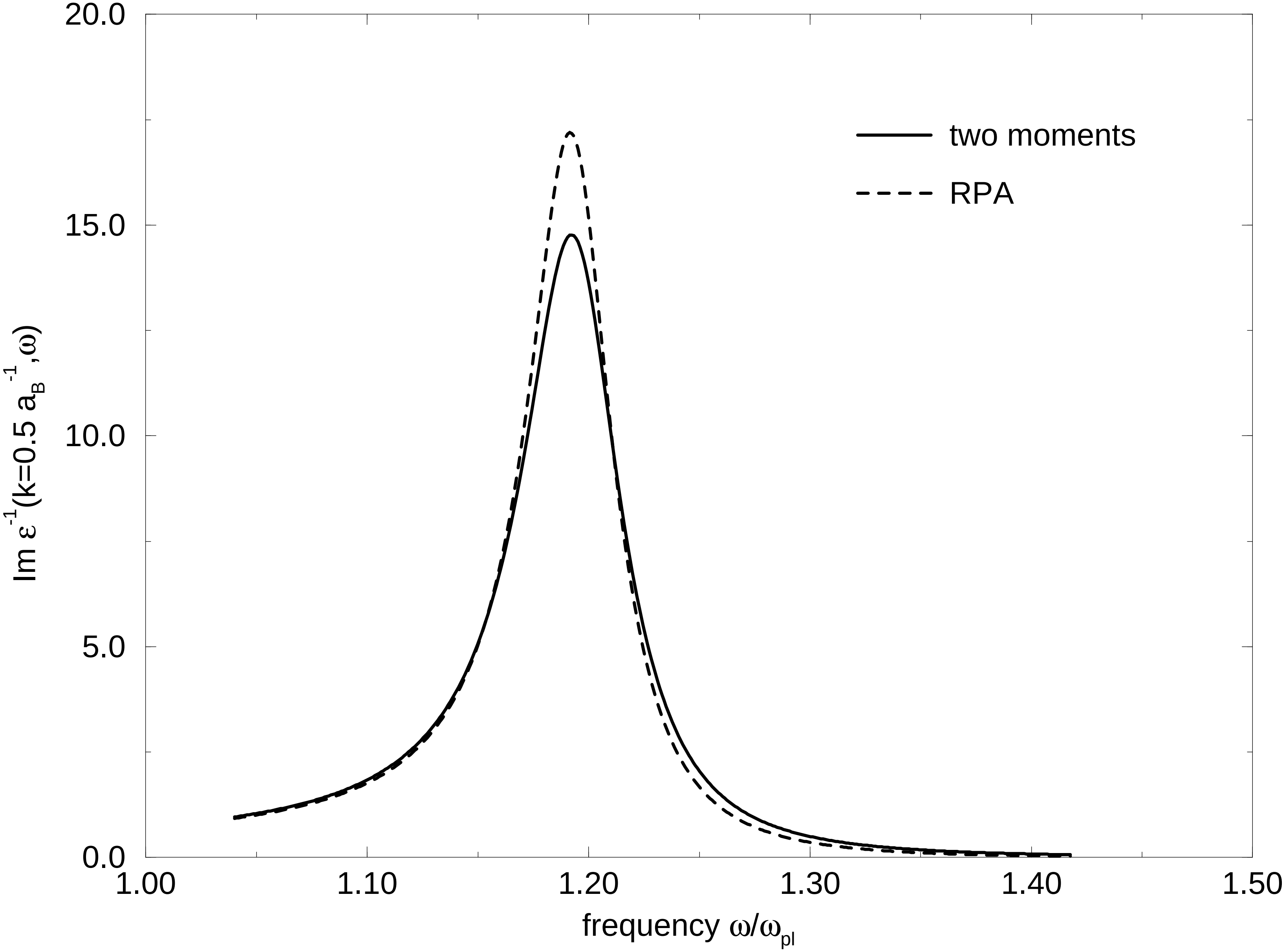
600.0
400.0
200.0
0.0
-200.0
Re ε(k=0.1 a_B^-1,ω)
two moments
RPA
one moment
10^-6
10^-4
10^-2
10^0
10^2
frequency ω/ω_pl

Fig. 2b



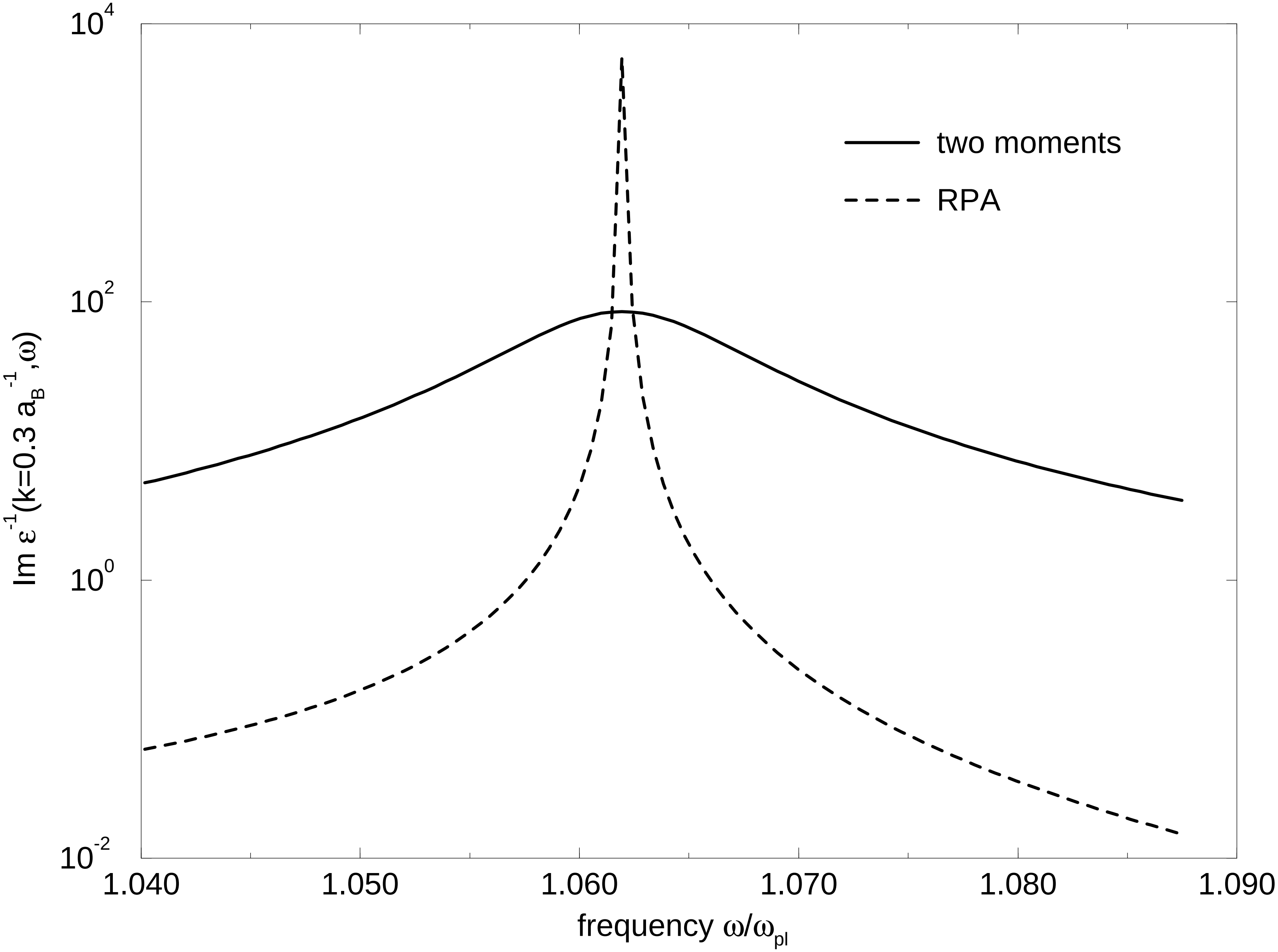
200.0
150.0
100.0
50.0
0.0
two moments
RPA
one moment
Im ε(k=0.1 a_B^-1,ω)
10^-6
10^-4
10^-2
10^0
10^2
frequency ω/ω_pl

Fig. 3



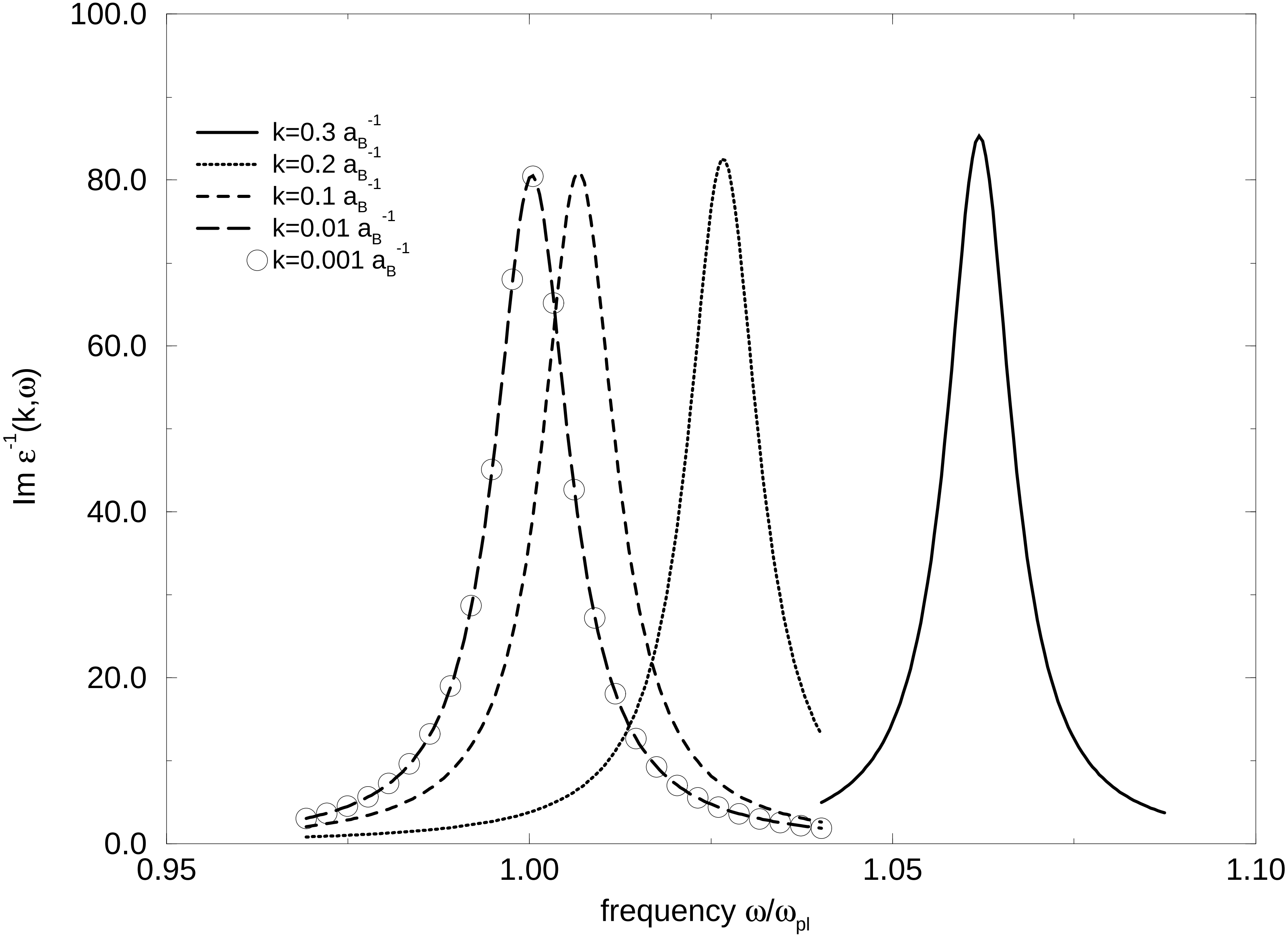
two moments
RPA
20.0
15.0
10.0
5.0
0.0
1.00
1.10
1.20
1.30
1.40
1.50
frequency ω/ω$_{pl}$
Im ε^{-1}(k=0.5 a$_B$$^{-1}$,ω)

Fig. 4



two moments
RPA
10^4
10^2
10^0
10^{-2}
1.040
1.050
1.060
1.070
1.080
1.090
frequency ω/ω_{pl}
$\mathrm{Im}\,\varepsilon^{-1}(k=0.3\,a_B^{-1},\omega)$

Fig. 5



k=0.3 a_B^-1
k=0.2 a_B^-1
k=0.1 a_B^-1
k=0.01 a_B^-1
k=0.001 a_B^-1
100.0
80.0
60.0
40.0
20.0
0.0
0.95
1.00
1.05
1.10
Im ε^-1(k,ω)
frequency ω/ω_pl